# THE TEMPERATURE SENSITIVITY OF COAXIAL AND SLABLINE RESONATORS

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A simple model of a slabline and coaxial resonator is developed which is useful for calculating the resonant frequency at ambient and other temperatures. A spreadsheet written to handle the calculations is described. Practical results are presented for comparison with theoretical predictions.

## 1 Introduction

Interdigital and combline filters, constructed from commonly available metallic components, such as brass, aluminium, steel etc, invariably are affected by their thermal expansion and contraction. The effect is especially significant in very narrow band filters where the change in the resonant frequency of slabline and coaxial resonators can result in a significant unwanted shift in the pass or stop band over an operational temperature range.

One obvious solution, and a necessary one for certain critical applications, is to manufacture all components from a material with an ultra low coefficient of expansion such as 'Invar', a steel-nickel alloy. For less critical applications the choice of the correct combination of metals for the housing, resonator and tuning screws can lead to a considerable reduction in temperature sensitivity.

The key to modelling temperature effects depends on having reasonably accurate expressions for the parallel plate and fringing capacitances which are needed to accurately predict the resonant frequency, given the relevant dimensions of the structure. Knowing the coefficients of expansion of the component parts, it is possible to calculate the modified dimensions and therefore the new resonant frequency at a changed temperature. A method of facilitating this task using a spreadsheet is demonstrated.

## 2 Analysis of the Resonant Frequency

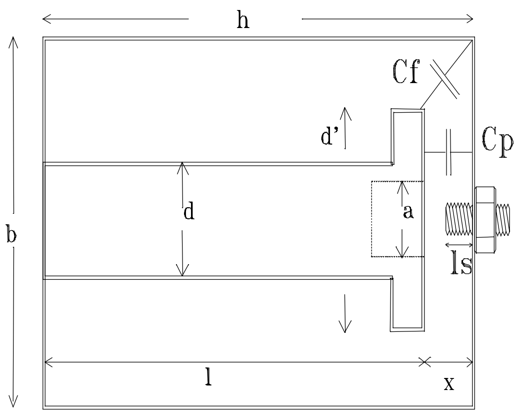



Fig1

Fig. 1 shows the cross section of either a coaxial or slabline resonator, of length l, and diameter d. The ground plane spacing, in the case of slabline, or cavity diameter, in the case of coax, is b. The fringing capacitance, parallel plate capacitance elements and screw capacitance elements are $C_f$, $C_p$ and $C_s$.

The loading capacitor necessary for resonance, in both coaxial and slabline resonators of electrical length θ, is:

$$C = \frac{1}{2\Pi f Z \tan\theta} \qquad (1)$$

which approximates to

$$C(pF) = \frac{159155}{f Z \tan\theta} \qquad \text{where f is in MHz} \qquad (2)$$

This loading capacitance is usually obtained from the sum of the parallel plate, the fringing and screw capacitances.

$$C = C_p + C_s + Cf \qquad (3)$$

Z is the resonator impedance given to a good approximation by:

$$Z = 60 \ln (b/d) \qquad \text{for a coaxial structure} \qquad (4)$$

or

$$Z = 60 \ln(4b/\Pi d) \qquad \text{for a slabline structure} \qquad (5)$$

The parallel plate capacitance may be calculated as:

$$Cp\ (pF) = \frac{0.00885\Pi d'^2}{4x} = \frac{0.00695\ d'^2}{x} \qquad (6)$$

where x is the gap (mm) between the end of the rod and the cavity.

Nicholson [1], presents a graph, from data attributed to Whinnery et al [2] showing the fringing capacitance of a slabline rod which can be approximated over the range d/b < 0.5 as:

$$C_f(pF) = 2.75 \times 10^{-2}\ d'\ (pF) \qquad (7)$$

where d′ is the resonator end diameter in mm.

Somlo [3] also gives plots for coaxial lines with step discontinuities. In the limiting case of the step being an open circuit, it is possible to approximate his curve as:

$$C_f(pF) = 0.0013b + 0.0164d' + 0.0183d'^2/b \qquad (8)$$

where b is the coaxial outer diameter in mm.

Both [2] and [3] treat the resonator as terminating in an open circuit without consideration of the presence of an end wall. So, the expressions would seem to be valid only for relatively large end gaps. An electrical field is present outside the 'parallel plates' of the resonator end wall, so there is always going to be some fringing capacitance and the assumption is made that the change from the open circuit value is small. The fringing capacitance when the gaps are small is only a small fraction of the total, so any uncertainty in the value should not be too serious a problem.

So we can write:

$$\text{resonant end gap (mm)} = \frac{0.00695d'^2}{C_p} = \frac{0.00695d'^2}{C - C_f - C_s} \qquad (9)$$

where d′ is the resonator end diameter in mm. & C is the required capacitance, in pF, for resonance at a frequency f. $C_f$ and $C_s$ are the fringing and screw capacitances.

If a 'counterbore' of diameter a is used on the end of the rod, the end capacitance is of course reduced. The parallel plate capacitance is reduced by an amount proportional to the cross sectional area of the counterbore but to some extent is offset by extra fringing capacitance. The exact calculation of this can be quite difficult and, at present, there is no known reliable source of data. A reasonable assumption might be that the effective cross sectional area is 90% of the real cross sectional area of the counterbore, if the counterbore diameter is less than 50% of the end diameter of the resonator.

Therefore:

$$\text{resonant end gap(mm)} = \frac{0.00695(d'^2 - 0.9 * a^2)}{C - C_s - C_f} \qquad (10)$$

C, the total capacitance required for resonance, is a function of frequency and all the dimensions of the resonant structure. Equating the physical end gap with the resonant end gap will lead to a solution of either frequency or any one of these dimensions.

## 3 Tuning screw capacitance

It is difficult to completely describe mathematically the behaviour of a tuning screw to include all fringing and parallel plate capacitances in the close proximity of a resonator end which may have a counterbore. However reasonable accuracy can still be obtained by choosing one of two possibilities:

1) The screw is approximately the diameter of the resonator and can be considered to be part of the cavity. Adjusting the depth of the tuning screw effectively varies the cavity height.

2) The screw diameter is smaller than the resonator and inserted into it forming a co-axial capacitor. The capacitance per unit length insertion can be calculated. The total crew capacitance is then just the product of insertion depth and the capacitance per unit depth of insertion.

## 4 Temperature Effects

If the dimensions of length above are defined at some temperature $T_0(^0K)$, then with a temperature rise of T the new dimension will be given by

$$M(T+T_0) = M(T_0)(1+ \alpha T) \tag{11}$$

M is any linear dimension and $\alpha$ is the coefficient of thermal linear expansion of the associated material.
The change in effective screw depth as a function of temperature can be expressed as:

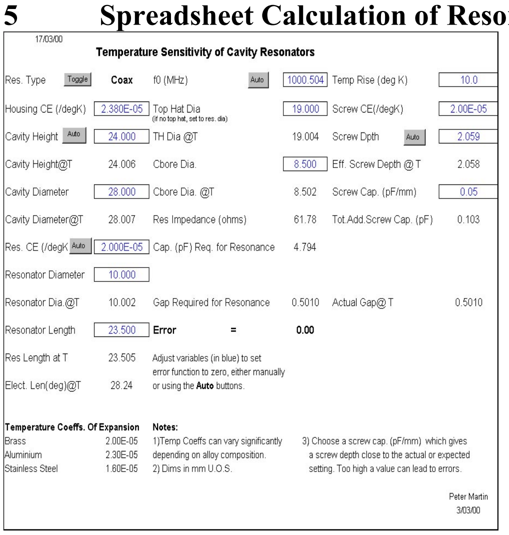
$$\Delta l_s = l\alpha_r + l_s\alpha_s - h\alpha_h \tag{12}$$

where $\alpha_r$, $\alpha_s$, $\alpha_h$ are the coefficients of linear expansion (CEs) of the resonator, tuning screw and housing respectively.

As the resonator expands with increasing temperature it may be expected that the resonant frequency will decrease. However the cavity itself will expand, changing the end gap. If the cavity expansion is greater than the resonator expansion, the decrease in capacitance will tend to produce a frequency increase. If the two effects are equal, the resonator will be temperature stable.

There are also lateral effects to consider. For instance, the diameter of the resonator will increase with temperature to increase the parallel plate capacitance and change the impedance of the resonator. Some of these changes are very small and likely to be negligible. However, the use of a spreadsheet to re-calculate all parameters at any temperature rise T enables all calculations to be performed quickly and no simplifying assumptions need to be made.
The inclusion of temperature effects adds the variables of CE and temperature which may be obtained by solving the equation of physical and resonant end gap.

## 5 Spreadsheet Calculation of Resonator Parameters



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**Temperature Sensitivity of Cavity Resonators**

| | | | | | |
|---|---|---|---|---|---|
| Res. Type [Toggle] | **Coax** | f0 (MHz) [Auto] | 1000.504 | Temp Rise (deg K) | 10.0 |
| Housing CE (/degK) | 2.380E-05 | Top Hat Dia (if no top hat, set to res. dia) | 19.000 | Screw CE(/degK) | 2.00E-05 |
| Cavity Height [Auto] | 24.000 | TH Dia @T | 19.004 | Screw Dpth [Auto] | 2.059 |
| Cavity Height@T | 24.006 | Cbore Dia. | 8.500 | Eff. Screw Depth @ T | 2.058 |
| Cavity Diameter | 28.000 | Cbore Dia. @T | 8.502 | Screw Cap. (pF/mm) | 0.05 |
| Cavity Diameter@T | 28.007 | Res Impedance (ohms) | 61.78 | Tot.Add.Screw Cap. (pF) | 0.103 |
| Res. CE (/degK [Auto] | 2.000E-05 | Cap. (pF) Req. for Resonance | 4.794 | | |
| Resonator Diameter | 10.000 | | | | |
| Resonator Dia.@T | 10.002 | Gap Required for Resonance | 0.5010 | Actual Gap@ T | 0.5010 |
| Resonator Length | 23.500 | **Error** = | **0.00** | | |
| Res Length at T | 23.505 | Adjust variables (in blue) to set error function to zero, either manually or using the **Auto** buttons. | | | |
| Elect. Len(deg)@T | 28.24 | | | | |

| **Temperature Coeffs. Of Expansion** | |
|---|---|
| Brass | 2.00E-05 |
| Aluminium | 2.30E-05 |
| Stainless Steel | 1.60E-05 |

**Notes:**
1)Temp Coeffs can vary significantly depending on alloy composition.
2) Dims in mm U.O.S.
3) Choose a screw cap. (pF/mm) which gives a screw depth close to the actual or expected setting. Too high a value can lead to errors.

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Fig 2

In the spreadsheet the actual gap is compared to the gap required for resonance and an error function is defined.(100 X the difference). Initially the temperature rise should be set to zero and the input variables, shown with their cells outlined, adjusted to set the error function to zero. Macros, attached to 'auto' buttons on the sheet, can be used to set the variables accurately.

If the behaviour of the tuning screw is initially not known, the resonator structure can be designed by setting $f_0$ around 10% -15% high and the tuning screw depth to zero. The frequency can later be tuned down to $f_0$ with the addition of a tuning screw capacitance.

The temperature sensitivity is determined by adding a temperature rise and re-solving for a new $f_0$.

The dimensions assume an air dielectric.

## Practical Results and Discussion of Accuracy of Model

| No | d(d') mm | b mm | h mm | l mm | a mm | Cav. Mat | Res. Mat | Scrw Mat. | Freq. theory MHz | Freq meas MHz | ΔF/ΔT theory kHz /degC | ΔF/ΔT meas kHz /degC |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 10 | 35 | 22 | 15.2 | 0 | Al(2) | Br | Br | | 3130 | -62 | -58 |
| 2 | 10 | 35 | 22 | 13.8 | 0 | Al(2) | Br | Br | | 3520 | -69.8 | -61 |
| 3 | 18 | 35 | 64 | 59.5 | 0 | Al(2) | Al(1) | Br | | 894 | -22.2 | -18 |
| 4 | 18 | 35 | 64 | 59.5 | 0 | Al(2) | Br | Br | | 890 | -13.1 | -9 |
| 5 | 10(19) | 28 | 24 | 23.5 | 8 | Al(2) | Br | Br | | 903 | 35.8 | 37.6 |
| 6 | 10(19) | 28 | 24 | 23.5 | 8 | Al(2) | Al (1) | Br | | 903 | -6.0 | -16.0 |
| 7 | 10(19) | 28 | 24 | 23.5 | 8 | Al(2) | Al(1) | None | 1000 | 1003 | -4 | -23.8 |
| 8 | 10(19) | 28 | 24 | 23.65 | 8 | Al(2) | Al(2) | None | 859 | 853 | -20 | -22.5 |

Table 1

Coefficients of Expansion
Al(1) $2.28 \times 10^{-5}$/°K
Al(2) $2.38 \times 10^{-5}$/°K
Brass $2.00 \times 10^{-5}$/°K

The resonators used in the tests were all of a coaxial type except for coupling apertures (in 1,2,3,4) milled into the side walls. These resonator structures can be described, and have an impedance, somewhere between axial and slabline.

The agreement between the theoretical and practical temperature coefficients as shown in Table 1 is reasonably good and within the limits of the uncertainty of the CEs.

It is important to obtain accurate figures of CE for the particular materials used. For instance, the combination of brass resonators and an aluminium body is a common choice and, with some adjustment of the gap, can give good temperature stability. However, brass can have a CE of between $1.8 \times 10^{-5}$ and $2.0 \times 10^{-5}$ and aluminium a CE between 2.2 and $2.4 \times 10^{-5}$. And so the difference can vary between 0.2 and $0.6 \times 10^{-5}$; figures which can produce markedly different temperature coefficients in the resonator.

This effect is shown in examples 6 and 7 which show the most disagreement. In these cases the gap between the resonator and end wall is small and the temperature performance is determined by the expansions of two similar but different materials. When the resonator and housing are made from the same material, as in example 8, the agreement is much improved.

The expressions used for the fringing and parallel plate capacitances present at the end of the resonator are the result of some assumption and approximation and could be improved if more data were available. Nevertheless the good agreement between the observed and measured resonant frequencies shows that they are still useful.

**References**

[1] B.F. Nicholson, "The Resonant Frequency of Interdigital Filter Elements", IEEE MTT-14, May 1966, pp 250 -251.

[2] J.R. Whinnery, H.W. Jamieson, and T.E. Robbins, "Coaxial Line Discontinuities", Proc I.R.E. vol32, Nov 1944 pp 695 – 709

[3] P.I Somlo, "The Computation of Coaxial Line Step Capacitances", IEEE MTT -15, Jan 1967,pp48-53.