Coupling Bandwidth and Reflected Group Delay Characterization of Microwave Bandpass Filters

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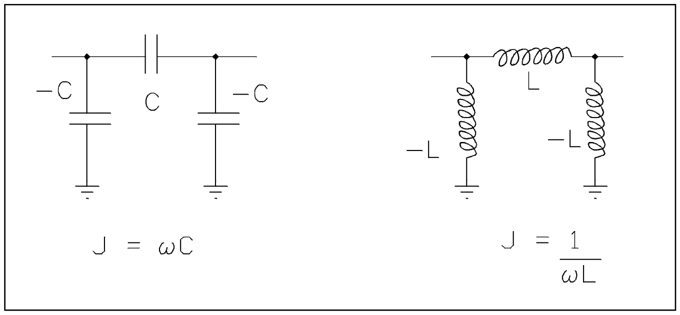
Bandpass filters, at least for the case of narrow and moderate bandwidths, may be uniquely defined by their Q, and their internal and external couplings. For instance, with tapped cavity coupled filters the tap point largely determines the external coupling and the size and proximity of the resonators determines the internal couplings.

This simple fact gives us a powerful technique for realizing filters that consist of arbitrary structures that are either not completely understood or are very difficult and time consuming to analyze fully. Providing that there is a rudimentary understanding of the couplings, the structures can be physically modified in the development stage to obtain the correct coupling bandwidths or reflected group delays and therefore the desired filter response. It is possible to measure coupling bandwidths and reflected group delays, which are directly related to each other, to a high degree of accuracy although the resonators may not be directly accessible within a filter. Even when filter couplings are relatively well understood, such as with interdigital and waveguide filters, there is often a need to empirically correct the structure for optimum response.

For narrow band filters the finite resonator Q is the limit for accuracy whereas for broader band filters the accuracy of the lowpass to bandpass transform and the frequency dependency of coupling networks and resonators are the limiting factors.

The techniques described in this article are generally applicable to all types of bandpass filters, lumped element, cavity resonator, dielectric resonator, waveguide, microstrip etc or filters with resonators which of different types.



▲ **Figure 1. Admittance inverters.**

Calculation of coupling bandwidths

The concept of coupling bandwidths is not new, Dishal [1,2] used it in the 1950s and ’60s to greatly simplify design procedures for interdigital filters. CBWs can be regarded as another way of expressing the admittance or impedance inverter values (*J* or *K*) but have the advantage of being directly and easily measurable in a filter using a Vector Network Analyzer (VNA).

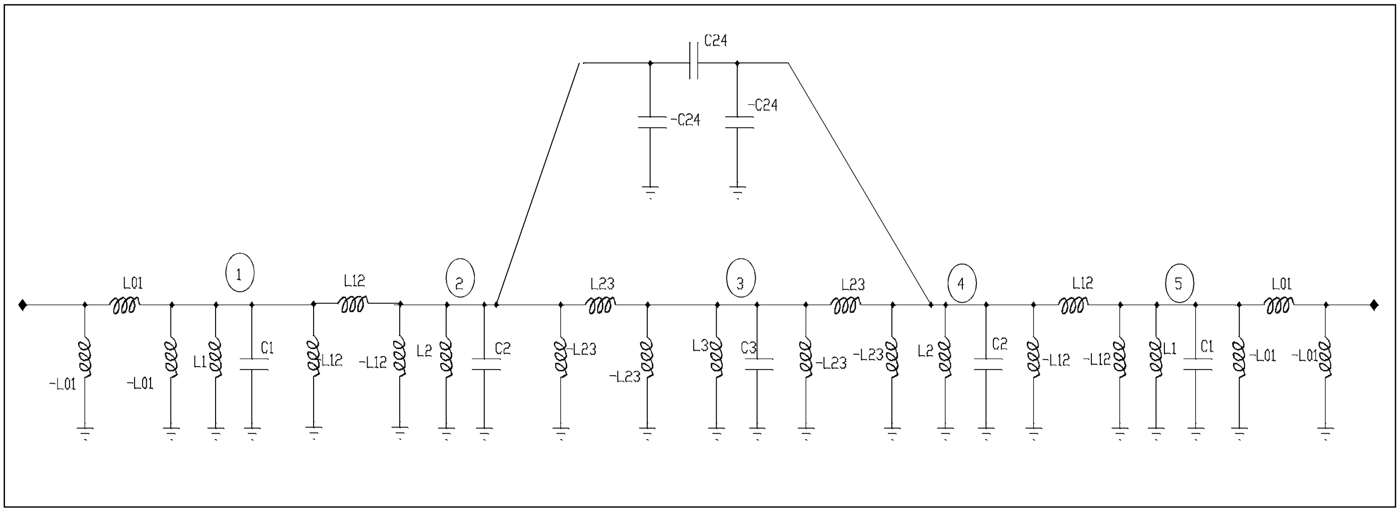
Normalized coupling bandwidths, CBWs, can be easily calculated from the lowpass prototype *g* values (usually Chebyschev). To calculate the normalized external couplings

$$q_1 = g_0\, g_1, \quad q_N = g_N\, g_{N+1} \qquad (1)$$

where usually $q_1 = q_N$. To calculate the internal couplings

$$k_{i,i+1} = 1/(g_i\, g_{i+1})^{1/2} \quad \text{for } i = 1 \text{ to } N–1 \qquad (2)$$

It is sometimes more descriptive to give these



▲ **Figure 2. A bandpass filter as parallel L-C components with admittance inverters.**

CBWs units of frequency and relate them to filter bandwidth as follows

$$K_{0,1} = BW/q_1 \text{ and } K_{N,N+1} = BW/q_N \quad \text{(for the external couplings)} \tag{3}$$

Usually $K_{0,1} = K_{N,N+1} = K_E$

$$K_{i,j} = k_{i,j}\, BW \quad \text{(for the internal couplings)} \tag{4}$$

A common requirement is for 20 dB return loss in a filter. It is advisable to aim for 25 dB and the g values, up to $N=7$ are shown in Table 1 for this case. For a method of calculating Chebyschev g values see [3], p. 99.

For example, a 939 MHz Chebyschev five element filter, with a bandwidth of 28 MHz and return loss of 25 dB requires coupling bandwidths of

K_E =	(in and out)		35.2 MHz
$K_{1,2}$ =	$K_{4,5}$	=	27.28 MHz
$K_{2,3}$ =	$K_{3,4}$	=	19.11 MHz

The Σg column can be used for calculating filter loss if the unloaded resonator Q is known. The loss [4] is given by

$$IL\ (\text{dB}) = (4.34/\text{BW} \times \text{Q})\ f_0\ \Sigma g \tag{5}$$

For the above filter a resonator Q of 2500 would lead to a loss of 0.34 dB in the centre of the pass band.

Relationship between coupling bandwidths, admittance inverters and lumped model equivalent circuit

As previously mentioned, coupling bandwidths are directly related to both admittance and impedance inverters which themselves can be used to eliminate either the inductive or capacitive elements in the lowpass prototype. Ideal inverters are frequency invariant, retaining their properties through a lowpass to bandpass transformation. A convenient way to understand a microwave bandpass filter is to consider the resonant elements as parallel L and C components, L_i, C_i etc., and the couplings as admittance inverters (Figure 2) consisting of inductive elements $L_{i,j}$ or Ci,j depending on whether the coupling is inductive or capacitive.

It is preferable, but not entirely necessary if the couplings are all in-line, to choose a circuit that approximates the actual coupling mechanism. For example, a combline filter has coupling predominantly via the magnetic field and an inductive inverter should be used, but in an interdigital filter where electric field coupling predominates a capacitive inverter should be chosen.

Couplings $J_{i,j}$ where $j = i+1$ are the main or in-line couplings and $J_{i,j}$ where $j = i+2$, $i+3$ etc. are cross couplings which can, optionally, be used to introduce interesting and useful perturbations to a standard response.

Figure 2 shows an inverter (inductive) coupled model of a 5 resonator filter, with a capacitive cross coupling between sections 2 and 4. The 5 resonators are repre-

N	Σg	g_0	g_1	g_2	g_3	g_4	g_5	g_6	g_7	g_8
1	0.11	1.000	0.1127	1.0000						
2	0.92	1.000	0.4881	0.4359	1.1192					
3	2.34	1.000	0.6703	1.0027	0.6703	1.0000				
4	4.02	1.000	0.7533	1.2252	1.3712	0.6731	1.1192			
5	5.86	1.000	0.7960	1.3248	1.6207	1.3248	0.7960	1.0000		
6	7.74	1.000	0.8205	1.3768	1.7285	1.5445	1.5409	0.7332	1.1192	
7	9.69	1.000	0.8358	1.4074	1.7843	1.6368	1.7843	1.4074	0.8358	1.0000

▲ **Table 1. *g* values for the example filter.**

sented by shunt LC elements.

The relationships between coupling parameters and inverter admittances follow. For the external couplings

$$J_{0,1} = J_{n-1,n} = \frac{\left(\pi K_{0,1} / 2f_0\right)^{1/2}}{Z_0} \qquad \left(K_{0,1} = K_{1,2} = K_E\right) \quad (6)$$

For the internal couplings

$$J_{i,j} = \frac{\pi K_{i,j}}{f_0 Z_0} \quad (7)$$

$$J_{0,1} = \frac{1}{2\pi f_0 L_{0,1}} = \frac{1}{2\pi f_0 L_{5,6}} \quad (8)$$

$$J_{i,j} = \frac{1}{2\pi f_0 L_{i,j}} \qquad \text{where } i = 2 \text{ to } 4$$

$$J_{2,4} = 2\pi f_0 C_{2,4} \quad (9)$$

Rearranging and simplifying (Z_0 = 50 ohms), we obtain

$$L_{0,1} = L_{5,6} = \frac{6.3494}{\left(K_E f_0\right)^{1/2}} \text{ nH} \qquad K_E \text{ and } f_0 \text{ are in GHz} \quad (10)$$

$$L_{i,j} = \frac{5.066}{K_{i,j}} \text{ nH} \qquad \text{where } K_{i,j} \text{ is in GHz} \quad (11)$$

$$C_{2,4} = \frac{5K_{2,4}}{K_{i,j}} \text{ pF} \qquad \text{where } K_E \text{ and } f_0 \text{ are in GHz} \quad (12)$$

For ideal resonators the capacitor and inductor values are typically set by

$$C_i = \frac{1}{4 f_{0i} Z_0} \qquad L_i = \frac{Z_0}{\pi^2 f_{0i}} \quad (13)$$

for i = 1 to n. f_{0i} is a frequency close to f_0. This can be simplified to

$$C_i = \frac{1}{4 f_{0i} Z_0} \qquad L_i = \frac{Z_0}{\pi^2 f_{0i}} \quad (14)$$

where f_{0i} is in GHz and for Z_0 = 50 ohms.

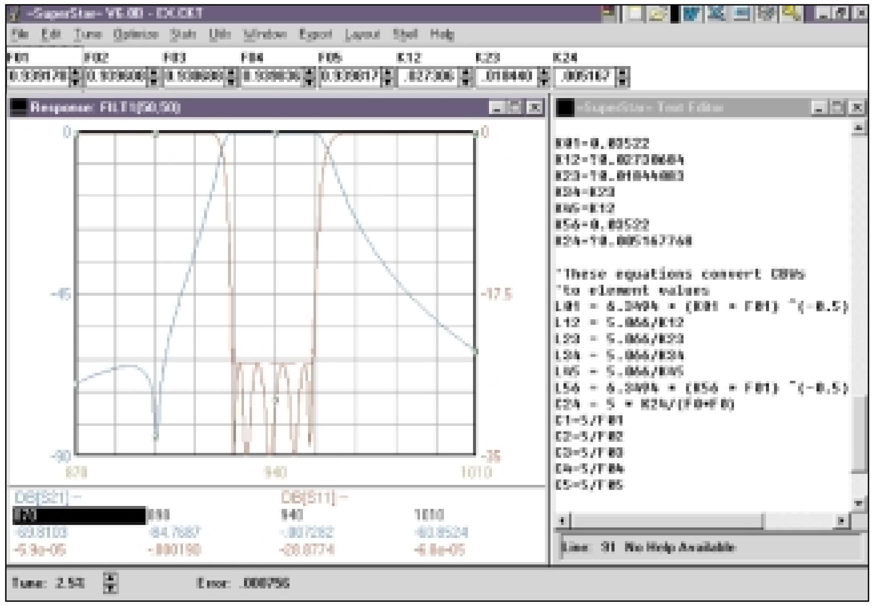
The above equations give a direct relationship between element values in the lumped model and coupling bandwidths. So, element values can be calculated from the coupling bandwidths and vice versa.

These equations can be programmed using a CAD package such as SuperStar [5]. A file, available from [6], written to describe the circuit of Figure 2 using the filter parameters of the above example is shown in Figure 3. First the CBWs as determined from the *g* values are put into the file and the cross coupling is set so low as to be negligible (i.e. <0.1 MHz). All the resonator frequencies, f_{0i}, are set to the center frequency of 939 MHz and a Chebyshev response is obtained as would be expected.

The cross coupling bandwidth can now be increased in steps and it will be noticed that a transmission zero will appear in the lower stopband. As the zero is brought closer to the passband the return loss in the passband will degrade and the optimizer will need to used to recover this. This may need to be done in more than one step if the cross coupling is very tight otherwise the optimizer will not find its way to a correct solution. Different optimization strategies may be tried but one that has been found to be successful involves keeping the external couplings at the initial calculated value. This stops the optimizer from finding a solution with a greater than wanted bandwidth, so long as the same return loss is obtained after optimization. Note that it is necessary to allow the resonant frequencies to vary, with cross coupling added, to obtain a good return loss. In practice this will be taken out by the tuning screws but the optimized CBWs will have to be used to achieve the correct filter response.

It is interesting to try replacing the capacitive cross coupling with an inductive coupling and also experiment with cross couplings across more than one resonator. Cross coupling can be used to introduce finite frequency, real (elliptic) or imaginary (linear phase) transmission zeros. Early examples of cross coupling in microwave filters have been described by Kurzrok. [7,8].

Caution needs to be exercised in the interpretation of the out of band response, which may be slightly different in the physical filter. For instance, with a waveguide filter, the rejection in the upper stopband will be somewhat less than predicted. The element values in the inverters are not of any particular significance, unless a lumped element filter is actually going to be constructed. The structure is best considered to be a purely mathematical model from which to obtain coupling bandwidths.



▲ **Figure 3. SuperStar is used to plot the response of the circuit.**

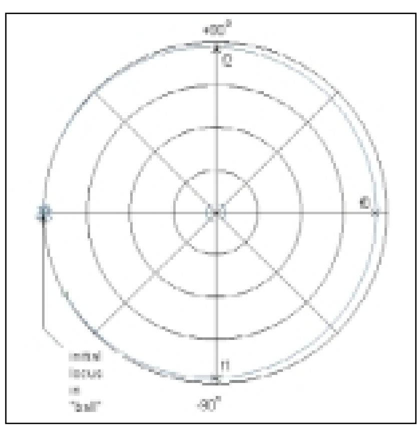
Measurement of coupling bandwidths

Atia and Williams [9] were the first to publish the method using the reflected phase to measure the coupling of microwave filters. This was later refined by Ness [10,11] and the method is as follows:

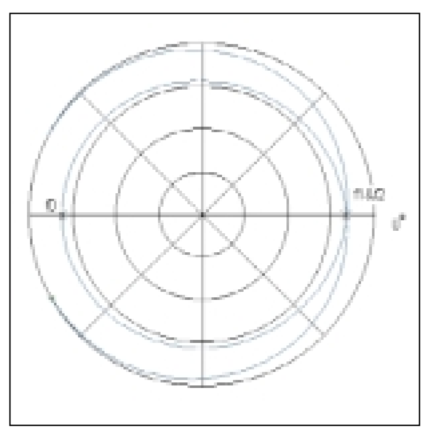
Calibrate a VNA for s_{11} in Smith Chart or polar display mode. Connect the input of the filter and short out all resonators. This can sometimes be done using the tuning screws or inserting a rod into the cavity. Adjust the delay or phase offset of the VNA to set the center frequency, which should be in a tight 'ball' at either the 0° or 180° points of the polar chart.

Remove the short from the first resonator and adjust its tuning screw to obtain a 180° phase shift of f_0. (See Figure 4). The locus will expand. The frequencies at points +90° and –90° are measured. K_E is given by:

$$K_E = f(+90°) - f(-90°) \qquad (16)$$



▲ **Figure 4. 180° phase shift tuning measurement.**



▲ **Figure 5. Tuning measurement for the second resonator.**

Repeat the process with the second resonator again producing a 180° change in f_0. Each time the locus expands and the 3 points where the locus of s_{11} crosses the real axis of the chart are measured. i.e. frequencies f_1, f_0, f_2. Note that these are now 180° points. (See Figure 5)

$$K_{12} = f_2 - f_1 \tag{17}$$

The third resonator is now restored and tuned to produce another 180° phase shift.

This time there are 5 crossing points, which are (in increasing frequency): f_2, f_1, f_0, f_3, f_4 (Note that f_1, f_2 etc are generally not the same as in the previous expression.)

$$K_{23} = f_3 - f_1 \tag{18}$$

Again the process is repeated with the fourth resonator to produce 7 crossing points: f_3, f_2, f_1, f_0, f_4, f_5, f_6.

$$K_{34} = [(f_4 - f_1)\,(f_6 - f_3)]K_{12} \tag{19}$$

The fifth resonator produces 9 crossing points: f_4, f_3, f_2, f_1, f_0, f_5, f_6, f_7, f_8

$$K_{45} = [(f_5 - f_1)\,(f_7 - f_3)]/K_{23} \tag{20}$$

The equations do get more complicated as the process is continued. However these are only needed occasionally and the references, particularly [11], may be consulted.

This method is also useful for tuning. The resonators can be brought in one at a time and set to the correct frequency. In practice it is usually easier to tune the last resonator with the VNA in transmission mode to get the best shape for s_{11} and s_{21}. The "last resonator" can be situated in the middle of the filter if the procedure is applied from both ends.

Measurement of reflective group delays

The group delay of s_{11} or s_{22} can also be used to characterise microwave filters. Again the method involves shorting all resonators and then consecutively restoring and tuning each one. The group delay is measured at each stage instead of the frequency crossing points.

For the first resonator, which, in the case of a directly coupled filter is also a transformer, the measurement enables the coupling bandwidth K_E, and from this, the correct tapping point to be calculated. This is already well known. However Ness [11] has shown that the continuation of this process yields meaningful results.

For the first resonator

$$K_E = 2/\pi t_{d1} \tag{21}$$

where t_{d1} is the measured group delay of s_{11} at f_0. See Figure 6. This can be expressed as

$$K_E \text{ (MHz)} = 636.6/t_{d1} \quad (22)$$

where t_{d1} is measured in ns. Again the tuning screw is used to move s_{11} through 180° each time a resonator is restored.

For the second resonator (see Figure 7)

$$K_{12} \text{ (MHz)} = 636.6/(t_{d1}t_{d2})^{1/2} \quad (23)$$

For the third resonator (see Figure 8)

$$K_{2,3}(\text{MHz}) = \frac{636.6}{\left[t_{d2}\left(t_{d3} - t_{d1}\right)\right]^{1/2}} \quad (24)$$

For the fourth resonator

$$K_{3,4}(\text{MHz}) = \frac{636.6}{\left[\left(t_{d3} - t_{d1}\right)\left(t_{d4} - t_{d2}\right)\right]^{1/2}} \quad (25)$$

For the fifth resonator

$$K_{4,5}(\text{MHz}) = \frac{636.6}{\left[\left(t_{d4} - t_{d2}\right)\left(t_{d5} - t_{d3}\right)\right]^{1/2}} \quad (26)$$

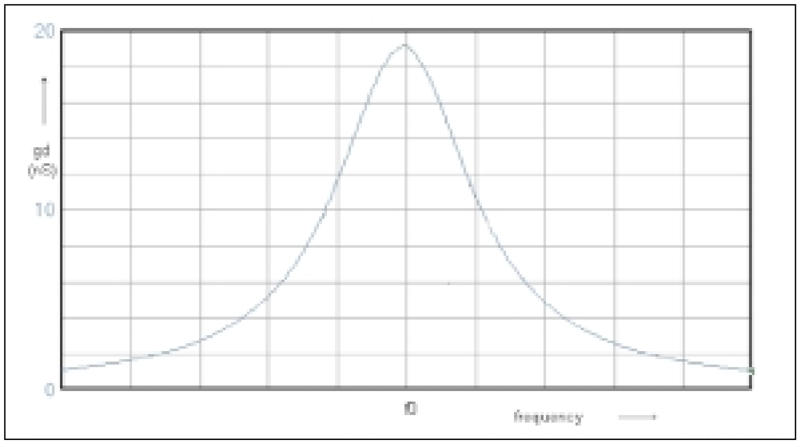
For the sixth resonator

$$K_{5,6}(\text{MHz}) = \frac{636.6}{\left[\left(t_{d5} - t_{d3}\right)\left(t_{d6} - t_{d4}\right)\right]^{1/2}} \quad (27)$$

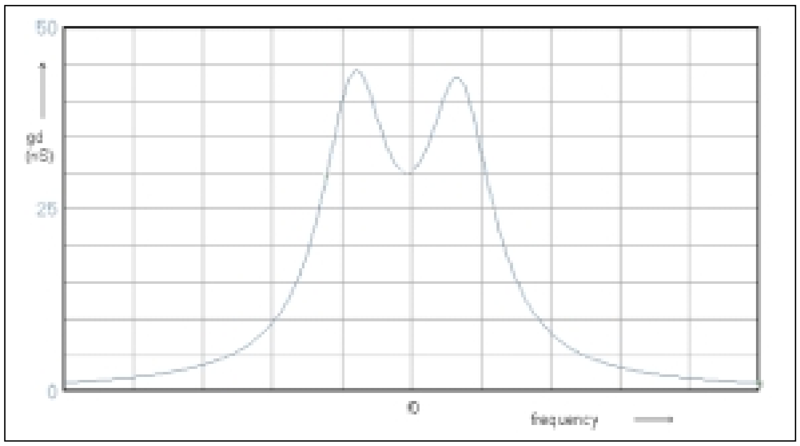
where t_{dn} (ns) is the group delay at f_0 of the nth resonator.

This method is particularly useful at the development stage. If the desired coupling bandwidths are known, the group delay at each stage can be calculated simply from the above formulae. Coupling structures can then be directly adjusted to obtain the correct value of t_d. It is probably easier to adjust for correct delays rather than correct frequency crossing points.

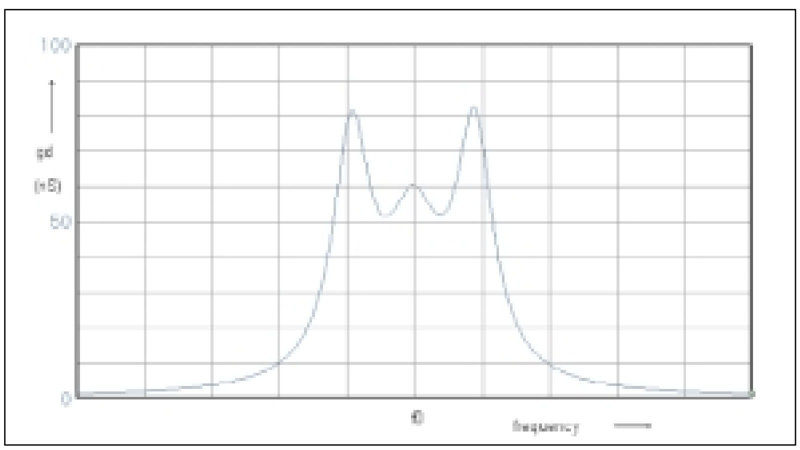
As [11] points out, it is just as valid to specify a filter in terms of its input delays as its coupling bandwidths. Some filter design software will generate plots of input group delay which can be directly compared with measurements taken on a VNA for alignment purposes.



▲ **Figure 6. Group delay: first resonator tuning.**



▲ **Figure 7. Group delay: second resonator tuning.**



▲ **Figure 8. Group delay: third resonator tuning.**

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